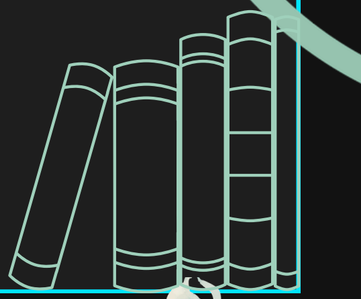
LINEAR EQUATIONS & SLOPES

Mastering Coordinates, Graphs, & Forms



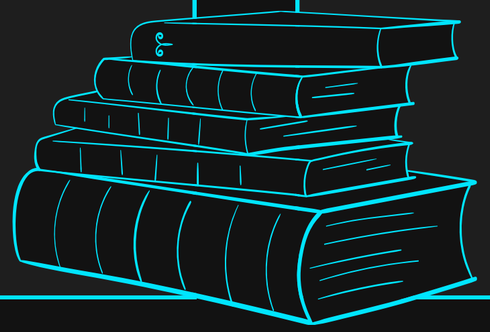
Finding Slope from Two Points

The Core Formula

- Slope (m) measures line steepness.
- Formula: m = (y2 - y1) / (x2 - x1)
- Always subtract coordinates in the same order.

Worked Example

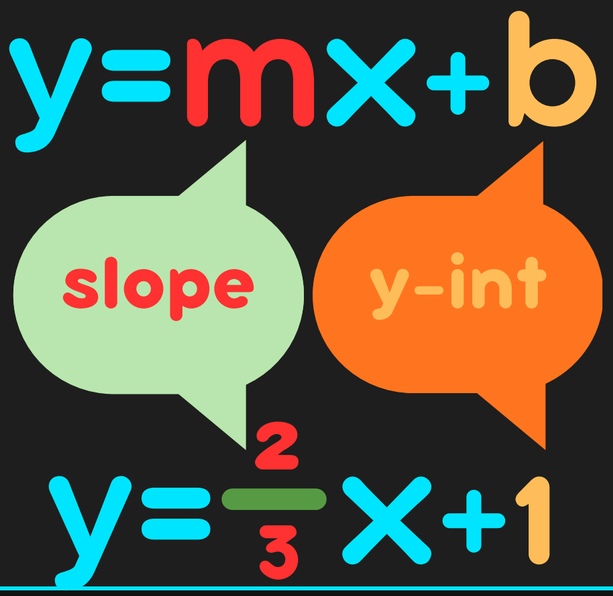
- Points: (5, -2) and (7, 9)
- Setup: m = (9 - (-2)) / (7 - 5)
- Simplify: m = (9 + 2) / 2
- Solution: m = 11 / 2



Finding Slope from Equations $y = mx + b$

Method 1: Isolate 'y'

- Rearrange into Slope-Intercept form: y = mx + b
- The slope (m) is the coefficient of x.
- Example: y = 2 - 5x rearrange to y = -5x + 2
- Result: m = -5



Notice

- Use when in Standard Form: Ax + By = C
- Formula shortcut: m = A / (opposite of B)
- Example: 5x + 2y = 14
- Shortcut setup: m = 5 / (-2)
- Result: m = -5/2

$Ax+By=C$

$y=\frac{2}{3}x+1$

$-\frac{2}{3}x+y=1$

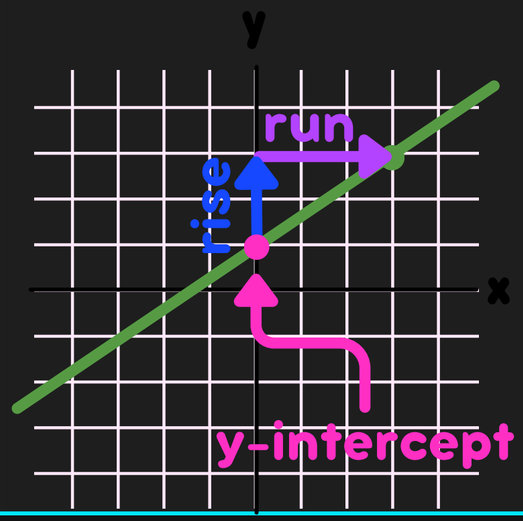
$-2x+3y=3$

Finding Slope from Graphs

Two Paths to Success

• Way 1 (Algebraic): Pick any two perfect grid points and use the coordinate formula.

• Way 2 (Visual): Draw a right triangle on the grid and count boxes directly.



The Geometric Shortcut

• Formula: m = Rise / Run

• Up-Right slant = Positive slope (+)

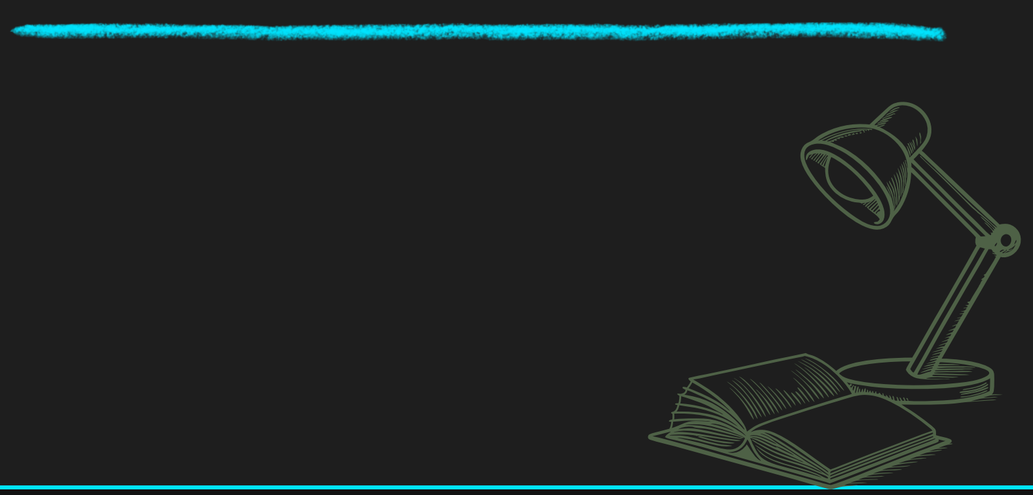
• Down-Right slant = Negative slope (-)

• Crucial step: Always double check the numeric value per grid box scale!

Special Cases — No 'x' or No 'y'

Horizontal Lines (No 'x')

- Equation form: y = constant (e.g., y = -5)
- Graph step: Draw a flat horizontal line through your point on the y-axis.
- Slope rule: Always m = 0



Vertical Lines (No 'y')

- Equation form: x = constant (e.g., x = 2)
- Graph step: Draw a straight vertical line through your point on the x-axis.
- Slope rule: Always m = Undefined

Finding Intercepts

X-Intercept (Ans, 0)

- Concept: Where the line crosses the horizontal x-axis.
- Step 1: Replace 'y' with zero.
- Step 2: Solve the remaining expression for 'x'.

Y-Intercept (0, Ans)

- Concept: Where the line crosses the vertical y-axis.
- Step 1: Replace 'x' with zero.
- Step 2: Solve the remaining expression for 'y'.
- Pro-Tip: Keep these calculations completely separate!

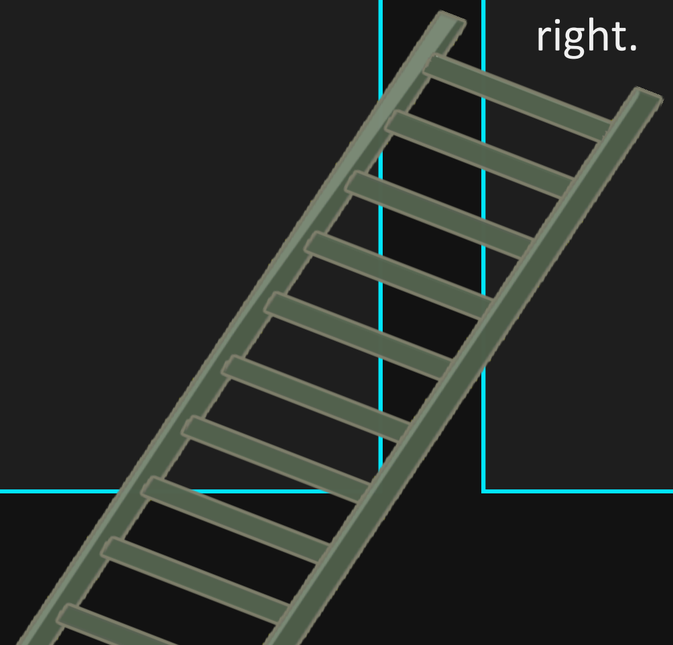
Graphing Linear Equations

Setup Checklist

- Isolate 'y' first into y = mx + b form.
- If the slope is a negative fraction, move the negative sign cleanly into the top numerator.
- This helps ensure clear movement directions.

Plotting Steps

- Step 1: Place your first dot directly on the y-intercept value (b).
- Step 2: Move from that start dot using your slope rise/run properties.
- Example: m = -3/5 means move 3 down and 5 right.



Writing Linear Equations

The Conversion Process

- Step 1: Find the slope (m) if not explicitly given.
- Step 2: Use point-slope form: y - y1 = m(x - x1)
- Change the x1, y1, and m variables only.

Clearing Fractions

- If the slope is a fraction N/D:
- Multiply the complete left side by Denominator (D).
- Multiply the right bracket by Numerator (N).
- Shift numbers to fit required form constraints.

Standard vs. Slope-Intercept Forms

Slope-Intercept Form

- Structure: y = mx + b
- Rule: 'y' must be completely by itself with no coefficient.
- Action: Divide everything by the original number attached to y.

$$y = mx + b$$

Standard Form

- Structure: Ax + By = C
- Rule 1: No fractions allowed anywhere.
- Rule 2: Leading term 'A' must be positive.
- Action: Move the 'x' term to the left side.

$$ax + by = c$$

System Line Relationships

Parallel Lines (||)

- Concept: Lines travel side by side and never meet.
- Slope Rule: Use the exact same slope value.
- Example: If line 1 has m = 3, line 2 has m = 3.

Perpendicular Lines (|_)

- Concept: Lines cross at a perfect 90 degree corner.
- Slope Rule: Flip the fraction and invert the sign (Negative Reciprocal).
- Example: m = -3/4 becomes perpendicular m = 4/3.

Student Practice — Set A

Question 1: Finding Slope

• Problem: Find the slope of the line passing through points (2, 3) and (4, 7).

• Goal: Apply the coordinate slope formula to calculate m.

Question 2: Finding Intercepts

• Problem: Find the x-intercept and y-intercept for $4x + 5y = 20$.

• Goal: Use zero substitutions to solve for both discrete coordinate points.

Student Practice — Set B

Question 3: Line Properties

• Problem: An initial base line possesses a slope value of -3/4.
• Goal 1: Determine the target slope of a parallel line.
• Goal 2: Determine the target slope of a perpendicular line.

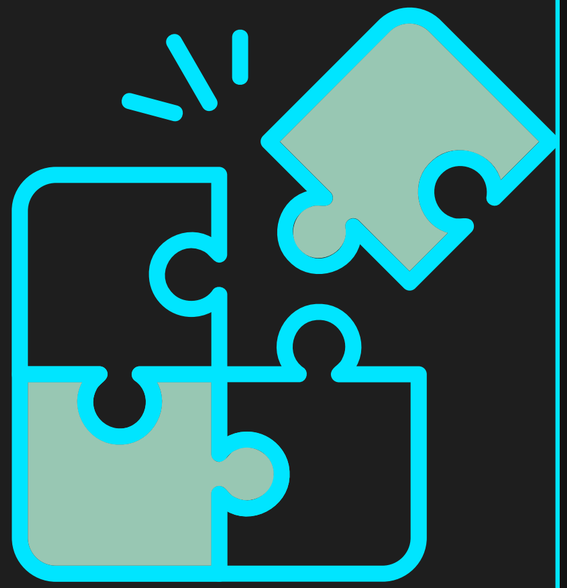
Question 4: Special Cases

• Problem: Identify the exact slope value and graphic direction for:
• Equation A: x = -5
• Equation B: y = 4

Step-by-Step Solutions — Set A

Solution 1 (Slope Calculation)

- Formula: m = (y2 - y1) / (x2 - x1)
- Setup: m = (7 - 3) / (4 - 2)
- Simplify: m = 4 / 2
- Final Answer: m = 2



Solution 2 (Intercept Calculation)

- X-Intercept: Set y=0 -> 4x = 20 -> x = 5.
- Coordinate Point: (5, 0)
- Y-Intercept: Set x=0 -> 5y = 20 -> y = 4.
- Coordinate Point: (0, 4)

Step-by-Step Solutions — Set B

Solution 3 (System Slopes)

- Parallel Rule: Slopes must match exactly.
- Parallel Answer: m = -3/4
- Perpendicular Rule: Invert fraction and switch signs.
- Perpendicular Answer: m = 4/3

Solution 4 (Special Case Output)

- Equation x = -5:
- Vertical Line through x-axis. Slope = Undefined.
- Equation y = 4:
- Horizontal Line through y-axis. Slope = 0.

Common Mistakes to Avoid

Formula & Arithmetic Traps

• Mixing X and Y: Don't put change in X over change in Y. It's always Rise over Run (Delta Y / Delta X).
• Sign Swaps: Be careful with negative values! m = (9 - (-2)) transforms into addition (9 + 2 = 11).
• Misordering: If you start with point 2 on top, you must start with point 2 on the bottom.

Form & Relationship Errors

• Standard Form Violations: Leaving fractions or letting 'A' stay negative is invalid standard form.
• Perpendicular Slip-ups: Forgetting to apply BOTH changes. You must flip the fraction AND switch the sign.
• Confusing Special Lines: Horizontal lines are y = constant (m=0). Vertical lines are x = constant (m=undefined).